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LETTERS TO THE EDITOR

ON THE NATURAL FREQUENCY OF A RECTANGULAR ISOTROPIC PLATE IN CONTACT WITH FLUID

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1. INTRODUCTION

In this paper, the free vibration of a rectangular isotropic plate in contact with fluid is investigated. It is well known that the fluid motion is influenced by the plate vibration and generates an important increase of kinetic energy of the whole system. Therefore, the natural frequencies of the plate in contact with the fluid can be determined by calculating the added virtual mass incremental factor (AVMI factor) which represents the kinetic energy due to the fluid. Lamb [1] calculated the change in natural frequency of a thin circular plate fixed along its boundary and placed in the aperture of an infinitely rigid wall in contact with water, then Powell and Roberts [2] verified the theoretical results of Lamb's work by conducting experiments. All the work mentioned above assumed the boundary condition that the outside of the circular plates is an infinitely rigid wall which cannot be applied to circular plates immersed in water or placed on a free surface since the outside boundary conditions differ from the rigid-wall condition. Kwak and Kim [3, 4] investigated the above problem and solved the mixed boundary problem by using Hankel transform. The purpose of this study is to calculate the natural frequencies of a rectangular isotropic plate for general boundary conditions, which is in contact with fluid.

2. PROBLEM FORMULATION

Consider the physical model of an isotropic rectangular plate in contact with fluid stated as in Figure 1, where $2a$ and $2b$ represent the width and length of the rectangular plate, and h is the thickness respectively. F denotes the fluid domain, S_1 denotes the surface between the fluid and an infinite rigid wall and S_2 denotes the surface between the fluid and the plate, also S_∞ denotes the surface at infinity.

The governing equation of the free vibration of a rectangular isotropic plate in contact with fluid neglecting the effects of rotatory inertia and transverse shear deformation can be written as follows:

$$D(\nabla^4 w) + (\rho_P h + M_f)\frac{\partial^2 w}{\partial t^2} = 0, \tag{1}$$

where w is the transverse deflection of the plate, D is the bending stiffness coefficient, ρ_P is the mass density of the plate h is the thickness of the plate and M_f denotes the fluid-added mass.

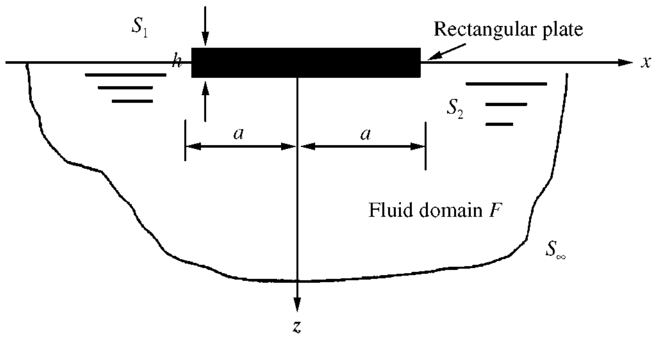

Figure 1. The rectangular plate in contact with fluid.

For undamped free-vibration analysis in the air, equation (1) becomes

$$D(\nabla^4 w) + \rho_P h \frac{\partial^2 w}{\partial t^2} = 0. \tag{2}$$

The solution of equation (2) can be obtained by using separation of variables and is represented by the following form:

$$\begin{aligned} w(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} X_m(x) Y_n(y) T(t) \\ &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} W(x, y) T(t), \end{aligned} \tag{3}$$

where

$$T(t) = \sin \omega t \tag{4}$$

and $X_m(x)$, $Y_n(y)$ are the orthogonal mode shape functions which satisfy the boundary conditions in the x and y directions respectively. It is quite well known that the natural frequency ω can be determined from the boundary conditions of the plate.

Now consider the rectangular plate that is in contact with liquid on one side only, the fluid is assumed to be incompressible and inviscid respectively. The fluid flow is considered as irrotational under plate vibration only so that its velocity potential can be expressed as

$$U(x, y, z, t) = \phi(x, y, z)\dot{T}(t), \tag{5}$$

where ϕ is the spatial distribution of the velocity potential and "•" denotes the derivative with respect to time.

Now it should be noted that ω is not the natural frequency of the plate in the air but rather the natural frequency of the plate in contact with the fluid. According to the assumption of the fluid, ϕ must satisfy the Laplace equation

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{in } F, \tag{6}$$

where F denotes the fluid domain. As in Figure 1, S_1 denotes the surface between the fluid and an infinite rigid wall and S_2 denotes the surface between the fluid and the plate, also S_∞ denotes the surface at infinity. The condition of the rigid wall on S_1, can be described as follows:

$$\left.\frac{\partial\phi(x, y, z)}{\partial z}\right|_{z=0} = 0 \quad \text{on } S_1. \tag{7}$$

Also, the interaction between the fluid and the plate can be represented by the following equation:

$$\left.\frac{\partial\phi(x, y, z)}{\partial z}\right|_{z=0} = -\bar{W}(x, y) \quad \text{on } S_2, \tag{8}$$

where $\bar{W}$ represents the "wet" mode shape of the plate vibrating in contact with the fluid. Furthermore, we must impose the conditions that the velocity potential ϕ and the velocities $\partial\phi/\partial x$, $\partial\phi/\partial y$ and $\partial\phi/\partial z$ approach zero on S_∞, i.e.,

$$\phi, \frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}, \rightarrow 0 \quad \text{as } x, y, z \rightarrow \infty \quad \text{on } S_\infty. \tag{9}$$

In this paper, the "wet" mode shape of the plate in contact with the fluid is assumed to be the same as the "dry" mode shape of the plate when vibrating in the air. The above assumption was verified by several researchers [5], therefore, the approximation $\bar{W}(x, y) = W(x, y)$ will be adopted in the following derivations.

Let us denote the double Fourier transform as follows:

$$\bar{\phi}(\xi, \eta, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi(x, y, z)\mathrm{e}^{+\mathrm{i}(\xi x+\eta y)}\,\mathrm{d}x\,\mathrm{d}y. \tag{10}$$

Applying double Fourier transform on equation (6) and using the boundary conditions specified in equation (9), then the velocity potential $\phi(x, y, z)$ can be expressed as

$$\begin{aligned}\phi(x, y, z) &= \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{\phi}(\xi, \eta, z)\mathrm{e}^{-\mathrm{i}(\xi x+\eta y)}\,\mathrm{d}\xi\,\mathrm{d}\eta \\ &= \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(\xi, \eta)\,\mathrm{e}^{-(\xi^2+\eta^2)^{\frac{1}{2}}z}\,\mathrm{e}^{-\mathrm{i}(\xi x+\eta y)}\,\mathrm{d}\xi\,\mathrm{d}\eta\end{aligned} \tag{11}$$

and $A(\xi, \eta)$ can be calculated as follows based on the boundary conditions in equations (7) and (8):

$$A(\xi, \eta) = (\xi^2+\eta^2)^{-1/2}\int_{-b}^{b}\int_{-a}^{a} X_m(x)Y_n(y)\mathrm{e}^{+\mathrm{i}(\xi x+\eta y)}\,\mathrm{d}x\mathrm{d}y. \tag{12}$$

It should be noted that generally $A(\xi, \eta)$ is a complex function of both ξ and η.

Based on the previous assumption that the wet mode shapes are almost equivalent to dry mode shapes, the natural frequency of the plate in contact with fluid ω_f can be determined from the following equation [3]:

$$\omega_{fmn} = \frac{\omega_{amn}}{\sqrt{1+\beta_{mn}}}, \tag{13}$$

where ω_{amn} is the natural frequency of the plate in the air and β_{mn} is the AVMI factor that denotes the ratio between the reference kinetic energy of fluid induced by the plate vibration and that of the plate which can be written as

$$\beta_{mn} = \frac{T_F}{T_P}. \tag{14}$$

The reference kinetic energy of the fluid can be obtained as follows from its boundary motion by using the assumption on the irrotational movement of the fluid flow:

$$T_F = -\frac{1}{2}\rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial\phi(x, y, 0)}{\partial z}\,\phi(x, y, 0)\,\mathrm{d}x\,\mathrm{d}y. \tag{15}$$

Substituting equations (7), (8) and (11) into equation (15) gives

$$T_F = \frac{1}{2}\left(\frac{1}{2\pi}\right)^2 \rho_F \int_{-b}^{b}\int_{-a}^{a}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_m(x)Y_n(y)A(\xi, \eta)\mathrm{e}^{-\mathrm{i}(\xi x + \eta y)}\,\mathrm{d}\xi\,\mathrm{d}\eta\,\mathrm{d}x\,\mathrm{d}y. \tag{16}$$

To evaluate the above multiple integral, reverse the order of integration, then equation (16) can be simplified to

$$\begin{aligned} T_F &= \frac{1}{2}\left(\frac{1}{2\pi}\right)^2 \rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{-b}^{b}\int_{-a}^{a} X_m(x)Y_n(y)\,\mathrm{e}^{-\mathrm{i}(\xi x + \eta y)}\,\mathrm{d}x\,\mathrm{d}y\right] A(\xi, \eta)\,\mathrm{d}\xi\,\mathrm{d}\eta \\ &= \frac{1}{2}\left(\frac{1}{2\pi}\right)^2 \rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [A^*(\xi, \eta)(\xi^2 + \eta^2)^{1/2}]\,[\mathrm{A}(\xi, \eta)]\,\mathrm{d}\xi\,\mathrm{d}\eta \\ &= \frac{1}{2}\left(\frac{1}{2\pi}\right)^2 \rho_F \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |A(\xi, \eta)|^2(\xi^2 + \eta^2)^{1/2}\,\mathrm{d}\xi\,\mathrm{d}\eta, \end{aligned} \tag{17}$$

where $*$ is the complex conjugate and $A(\xi, \eta)$ can always be determined from equation (12) provided that the dry mode shapes of the plate are known.

The reference kinetic energy of the plate can be obtained as

$$T_P = \frac{1}{2}\rho_P h \int_{-b}^{b}\int_{-a}^{a} X_m^2(x)Y_n^2(y)\,\mathrm{d}x\,\mathrm{d}y. \tag{18}$$

Therefore, β_{mn} (AVMI factor) can be evaluated from equation (14) after T_F and T_P are computed and eventually ω_{fmn} can be determined from equation (13).

3. NUMERICAL EXAMPLE

We now investigate β_{mn} (AVMI factor) of a rectangular isotropic plate in contact with fluid to illustrate the previous formulations. In this study, the following material parameters of a rectangular plate are selected as follows:

$$h = 0{\cdot}05\,\mathrm{m}, \quad \rho_P = 2{\cdot}44 \times 10^3\ \mathrm{kg/m^3}, \quad \rho_f = 1000\ \mathrm{kg/m^3}.$$

The units of length $2a$ and width $2b$ of the plate are specified in meters throughout all the numerical analysis.

Tables 1 and 2 present the values of added virtual mass incremental factor (AVMI factor) for a square plate with the simply supported boundary condition. Tables 3 and 4 show the values of a square plate with the clamped boundary condition. In these tables, we can detect that the value of β_{11} is much lager than those of the other modes for both boundary conditions. This means β_{11} plays a dominant role as far as the AVMI factor is considered; meanwhile we can see that β_{mn} decreases with the mode order. This implies that the effect of fluid also decreases with mode order. In general, AVMI factor β_{mn} of the higher mode number is smaller than that of the lower mode number because the fluid movement stroke

TABLE 1

The values of β_{mn} for odd mode in a simply supported square plate

β_{mn}	$m = 1$	$m = 3$	$m = 5$	$m = 7$
$n = 1$	6·9526	2·2540	1·3045	0·9124
$n = 3$	2·2540	1·3938	0·9835	0·7442
$n = 5$	1·3045	0·9835	0·7878	0·6409
$n = 7$	0·9124	0·7442	0·6409	0·5508

TABLE 2

The values of β_{mn} for even mode in a simply supported square plate

β_{mn}	$m = 2$	$m = 4$	$m = 6$	$m = 8$
$n = 2$	2·1696	1·3110	0·9105	0·6902
$n = 4$	1·3110	0·9939	0·7695	0·6165
$n = 6$	0·9105	0·7695	0·6449	0·5438
$n = 8$	0·6902	0·6165	0·5438	0·4775

TABLE 3

The values of β_{mn} for odd mode in a clamped square plate

β_{mn}	$m = 1$	$m = 3$	$m = 5$	$m = 7$
$n = 1$	5·9219	2·2449	1·3381	0·9433
$n = 3$	2·2449	1·3528	0·9657	0·7398
$n = 5$	1·3381	0·9657	0·7690	0·6289
$n = 7$	0·9433	0·7398	0·6289	0·5401

TABLE 4

The values of β_{mn} for even mode in a clamped square plate

β_{mn}	$m = 2$	$m = 4$	$m = 6$	$m = 8$
$n = 2$	1·9393	1·2467	0·8917	0·6862
$n = 4$	1·2467	0·9510	0·7461	0·6044
$n = 6$	0·8917	0·7461	0·6265	0·5313
$n = 8$	0·6862	0·6044	0·5313	0·4671

TABLE 5

The values of β_{11} for different plates with various length and width in the simply supported case

β_{11}	$a = 1$	$a = 2$	$a = 3$	$a = 4$	$a = 5$
$b = 1$	6·9526	9·6576	11·4467	12·8104	13·9284
$b = 2$	9·6576	14·2525	17·4782	20·0078	22·1128
$b = 3$	11·4467	17·4782	21·8967	25·4431	28·4376
$b = 4$	12·8104	20·0078	25·4431	29·8882	33·6886
$b = 5$	13·9284	22·1128	28·4376	33·6886	38·8859

TABLE 6

The values of β_{11} for different plates with various length and width in the clamped case

β_{11}	$a = 1$	$a = 2$	$a = 3$	$a = 4$	$a = 5$
$b = 1$	5·9219	8·2130	9·7213	10·8658	11·8002
$b = 2$	8·2130	12·0969	14·8124	16·9321	18·6890
$b = 3$	9·7213	14·8124	18·5249	21·4921	23·9877
$b = 4$	10·8658	16·9321	21·4921	25·2059	28·3688
$b = 5$	11·8002	18·6890	23·9877	28·3688	32·1399

TABLE 7

The comparison of β_{mn} for simply supported and clamped plates in a square isotropic plate

$a = b = 1$	Simply supported	Clamped
β_{11}	6·9526	5·9219
β_{22}	2·1696	1·9393
β_{33}	1·3938	1·3528
β_{44}	0·9939	0·9510
β_{55}	0·7878	0·7690
β_{66}	0·6449	0·6265
β_{77}	0·5508	0·5401
β_{88}	0·4775	0·4671

of the lower mode number is larger than that of the higher one. Therefore, as the mode number increases, the fluid movement stroke will be reduced which will result in the reduction of the added mass and AVMI factor. We can also obtain the similar phenomenon described in the free vibration of a liquid-filled circular cylindrical shell [6].

In Table 5, the values of β_{11} for different plates with various length and width in the simply supported plate are presented. Table 6 shows the values of β_{11} for different plates with various lengths and widths in the clamped case. As we can see, the values of β_{11} are symmetric about a and b. It is reasonable because the case of $a = 1$; $b = 2$ is absolutely the same as that of $a = 2$; $b = 1$ physically speaking. Also we can state that the value of β_{11} will be increased if we enlarge the width or length of the plate, besides, the large area of the plate will have the larger values of β_{11}.

Table 7 is the comparison of β_{nn} for simply supported and clamped plates. As expected, the AVMI factor for simply supported case is always larger than that for the clamped case in all modes.

4. CONCLUSIONS

In this paper, the free vibration of a rectangular isotropic plate in contact with the fluid is investigated. It is well known that the fluid motion is influenced by the plate vibration and generates a remarkable increase of kinetic energy of the whole system. Therefore, the natural frequencies of the plate in contact with the fluid can be determined by calculating the AVMI factor, which represents the kinetic energy due to the fluid. Furthermore, it has been verified experimentally that only small changes in the wet mode shapes occur under fluid movement which enable us to assume that the wet mode shapes are almost equivalent to the dry mode shapes. The proposed approach can be adopted as a guidance to the engineer who are engaged in the vibration analysis and the design of the rectangular isotropic plates coupled with fluid. Also it should be noted that the presented approach can be used for any arbitrary shapes of plate with general boundary conditions provided that the natural frequencies of the plate in the air has been calculated.

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